Mean Value Theorem Suppose that f is continuous on $[a,b]$ and differentiable on (a,b). Then there is some point c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

A function f is **strictly increasing** in an interval I, if for all x, y in I where $x < y$, $f(x) < f(y)$.

For each of the following statements, explain why they are true (citing appropriate theorems or definitions) or give a counterexample. In all statements, we assume that f is continuous on its domain $[a,b]$ and differentiable on (a,b).

1. If $f' > 0$ on (a,b), then $\frac{f(b)-f(a)}{b-a} > 0$.

2. If $\frac{f(b)-f(a)}{b-a} > 0$, then $f' > 0$ on (a,b).

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3. If $f' > 0$ on (a, b), then f is strictly increasing on (a, b).

4. If f is strictly increasing on (a, b), then $f' > 0$ on (a, b).

5. If $f' = 0$ on (a, b), then f is constant on (a, b).

6. If c is a point of inflection of f, then $f''(c) = 0$.

7. If $f''(c) = 0$, then c is a point of inflection of f.

8. If $f''(c) > 0$, then f has a local minimum at c.

9. If f has a local minimum at c, then $f''(c) > 0$.

Extra Problem

If you have time, try to prove the following remarkable fact: if f is differentiable, then f' has the *intermediate value property*, i.e., if a and b are in the domain of f and y is between $f'(a)$ and $f'(b)$, then there is x between a and b such that $f'(x) = y$.