Math 1110: **The First Derivative Test for Local Extrema** Oct 24 (Wed)

Created by S. Bennoun, M. Hin, and T. Holm ©, modified by Jun Le Goh

Last Friday we discussed how to find the global extrema of a continuous function on a closed interval. Today we discuss local extrema:

Let f be a function with domain D.

f has a *local (or relative) maximum* at a point $a \in D$ if $f(a) \geq f(x)$ for all x in some open interval containing a.

f has a *local (or relative) minimum* at $a \in D$ if:

If f has a local extremum at a point a, and f is differentiable at a, what can we say about $f'(a)$?

Give an example of a function f which has a local extremum at 0 but is not differentiable at 0.

Now we work towards the First Derivative Test. First, state the Mean Value Theorem:

Suppose that f is continuous on $[a,b]$ and differentiable on (a,b). Show that if $f'(x) > 0$ for all $x \in (a,b)$, then f is increasing on $[a,b]$. (Recall the definition of increasing: f is increasing on $[a,b]$ means that for all points x and y in $[a,b]$, if $x < y$, then $f(x) < f(y)$.)

Again, suppose that f is continuous on $[a,b]$ and differentiable on (a,b). Show that if $f'(x) < 0$ for all $x \in (a,b)$, then f is decreasing on $[a,b]$. (Hint: You could use MVT, or you could apply what you showed in the previous question to an appropriate function.)

Using the two facts you just showed, you can now derive the First Derivative Test:

> Let c be a critical point of f. Suppose that f is differentiable at every point in some interval containing c, except possibly at c. Then:
>
> - if $f'(x)$ changes from negative to positive as x moves from below c to above c, then:
>
> - if $f'(x)$ changes from positive to negative as x moves from below c to above c, then:
>
> - if $f'(x)$ has the same sign as x moves from below c to above c, then:

Find the critical points of $f(x) = x^3 - 12x - 5$.
Identify the intervals on which f is increasing and the intervals on which f is decreasing.
Use the First Derivative Test to identify the local maxima and minima of f.