Created by S. Bennoun, M. Hin, and T. Holm ©, modified by Yuwen Wang
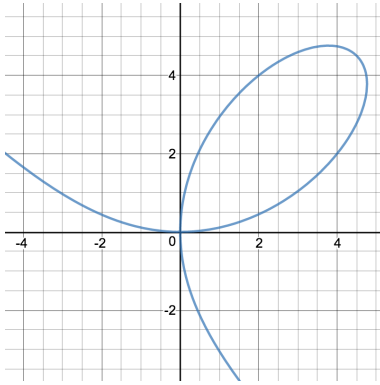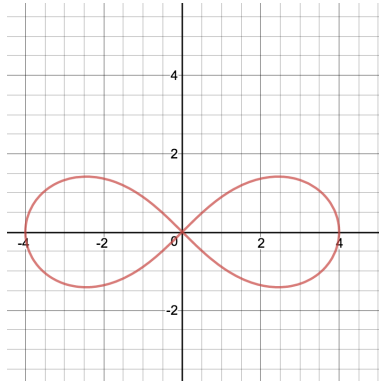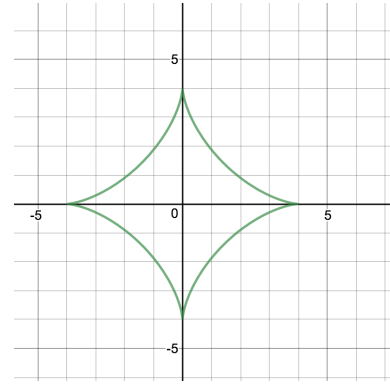
i) For each of the curves presented below, how many functions do we need to describe them and why? Moreover, can we describe any of them with a single function?



(a) Folium of Descartes
$x^3 + y^3 = 9xy$



(b) Lemniscate of Bernoulli
$x^4 + 2x^2y^2 + y^4 = 16(x^2 - y^2)$



(c) Astroid
$x^{2/3} + y^{2/3} = 4^{2/3}$

ii) Using the chain rule (that we studied last time), what is the derivative of the function $(3\sin x + 4x)^2$?

iii) Let us suppose we now decide to set $y = y(x) = 3\sin x + 4x$. Rewrite the above function and its derivative using only $y$ and $y'$.

iv) Compute the derivative of $\cos(x^3 + 2x + 5)$.

v) Let us set $y = y(x) = x^3 + 2x + 5$. Rewrite the above function and its derivative using $y$ and $y'$ but not $x$.