Math 1110: **Rates of Change** Sept 26 (Wed)

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1. Water is being poured into a cylindrical vase of radius 2 at a rate of 3 m^3/s. What is the rate of change of the water level with respect to the volume of water in the vase?

2. It costs CTB

$$C(x) = 100 + 2x - 0.005x^2, \quad 0 \leq x \leq 200$$

   dollars to bake x bagels.

   (a) Without directly calculating $C(101) - C(100)$, calculate an estimate for how much it cost to bake the 101^{st} bagel. (In economics, this is called the *marginal cost.*)

   (b) How about the 151^{st} bagel? Before estimating the cost, do you expect it to be lower or higher than the cost in (a)?

3. You have in front of you a cube and a ruler. You seek the volume of the cube, so you use the ruler to measure the side length of the cube. It is 2 units.

   (a) What is the volume of the cube?

   (b) It turns out that your ruler is only precise up to 0.1 units. What is the maximum error of your answer in (a), i.e., in the worst case, how much does it differ from the actual volume of the cube?

   (c) Your friend brings you a better ruler, which is precise up to 0.01 units. You measure the side length of the cube again and it still seems to be 2 units. In that case, what is the maximum error of your answer in (a)?

(d) What is the derivative of the volume V of a cube as a function of its side length? What is the value of $V'(2)$?

(e) How is (d) related to (b) and (c)? Explain this using limits.

4. The (basic) logistic model for population growth posits that the population as a function of time is

$$P(t) = \frac{KP_0e^t}{K + P_0(e^t - 1)},$$

where K is the carrying capacity and P_0 is the initial population.

(a) Calculate $\lim_{t\to\infty} P(t)$. What does this tell you about what the model predicts?

(b) What is the physical meaning of $P'(t)$?

(c) Actually, $P(t)$ is obtained by solving the *differential equation*

$$P'(t) = P\left(1 - \frac{P}{K}\right) = P - \frac{P^2}{K}.$$

Challenge problem: verify that P satisfies the above equation!