Math 1110: **Derivative of Trigonometric Function (3.5)** Sept 19 (Wed)

Created by S. Bennoun, M. Hin, and T. Holm ©, modified by Yuwen Wang

1. Objectives.

- compute the derivative of trigonometic functions $(\sin x, \cos x, \tan x)$

2. Compute the derivative of sin x.

(a) Writing the $\lim_{h\to 0}$ definition of the derivative, write down the definition of $f'(x) = (\sin x)'$.

(b) Let us compute this limit. To that end, we will need the trigonometric identity $\sin(u+v) = \sin u \cos v + \cos u \sin v$. We will moreover need compute $\lim_{h\to 0} \dfrac{\sin h}{h}$ and $\lim_{h\to 0} \dfrac{\cos h - 1}{h}$.

(c) So, the derivative of $\sin x$ is

3. Derivative of tangent.

Similarly to what we have done above, one can prove that $(\cos x)' = -\sin x$.
Using these two derivatives as well as the definition of $\tan x$, compute the derivative $(\tan x)'$.

4. Computing derivatives with trig functions.

Compute the derivatives of the following functions.

a) $f(t) = \sin t \cos t$

b) $g(x) = \sin x + e^x \cos x$

c) $h(z) = 2\tan z + \dfrac{3z}{\cos z}$

d) $f(x) = \sqrt{x} + \dfrac{\cos x}{\sin x} + 4$

5. Extra practice.

Does the graph of the function $f(t) = t + \cos t$ have a horizontal tangent line on the interval $[0, 2\pi]$? What about $g(t) = 2t + \cos t$?