Math 1110: **Limits at infinity** Sept 10 (Mon)

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1. Objectives.

- Definition of limits at infinity
- Compute various limits at infinity for rational functions, function horizontal and vertical asymptotes, functions where the behavior at inifinity is "$\infty - \infty$."

2. Definition of Limits at Infinity

We can think of the *limit of $f(x)$ as x approaches infinity* in the following way: choose *any* pre-determined level of precision. Then the limit $\lim_{x\to\infty} f(x)$ equals L if we can find a number $N > 0$ such that for any $x > N$ the function $f(x)$ approaches L with the desired pre-determined level of precision.

Note that the limit is still a *single number.*

3. Practice problems.

1. Compute the limits as x goes to infinity and negative infinity for the following functions:

   (a) $f(x) = \dfrac{2x^4 + 2x^2 - 3}{3x^4 + x^3 - 2x^2}$

   (b) $g(x) = \dfrac{2x^5 + 2x^2 - 3}{3x^4 + x^3 - 2x^2}$

   (c) $h(x) = \dfrac{2x^4 + 2x^2 - 3}{3x^6 + x^3 - 2x^2}$

   (d) $j(x) = \dfrac{\sqrt{2x^6 + 2x^2 - 3}}{3x^3 - 2x^2}$

2. Compute the horizontal asymptotes of $f(x) = \dfrac{\sqrt[3]{x} - 4x + 7}{3x + x^{2/3} - 1}$

3. Compute the horizontal asymptotes of $g(x) = \dfrac{1}{x}\sin x$ (compare your answer with $\lim_{x\to\infty} \sin(\pi x)$ that you computed in the pre-class activity).

4. Determine the vertical and horizontal asymptotes of the following functions:

   (a) $f(x) = \dfrac{2x^2 + 1}{3x - 5}$

   (b) $f(x) = \dfrac{2x^2 + 5}{x^2 - 5x}$

4. Extra practice.

1. Compute $\lim_{x\to\infty}(x^2 - x)$.

2. Compute $\lim_{x\to\infty}(\sqrt{x^2+2} - x)$.

3. Compute $\lim_{x\to\infty}(\sqrt{x^2+ax} - \sqrt{x^2+bx})$, where a and b are constants

4. Compute the horizontal asymptote of $f(x) = \dfrac{-x^2+5x-1}{2x+3}$.